From Eratosthenes to Global Positioning Systems: Calculating the Size of the Earth

Geology 1P — Mr. Traeger

Name: Key   Period: ______   Date: ______

Purpose and Background

Eratosthenes was a Greek mathematician who lived between 276 and 195 BC. He is best known for calculating the circumference of the Earth and the axial tilt of the Earth with a good degree of accuracy. Eratosthenes calculated the circumference of the Earth on June 21st (The Summer Solstice) by noting that the Sun's rays (assumed to be parallel everywhere on the Earth from a distance of 93 million miles) shone in to a well at noon in Alexandria, Egypt at a slightly different angle than a well in Syene, Egypt. The Sun shone into a well in Alexandria at about a 7° angle from the vertical and the Sun shone into a well from directly overhead (0° angle from the vertical) in Syene. On June 21st, the Sun will shine down on Earth from directly overhead at places that are 23.5° N latitude (Tropic of Cancer). Syene is near the Tropic of Cancer, so the Sun will be directly overhead at noon on the Summer Solstice on June 21st. We are going to duplicate Eratosthenes' experiment by using a GPS receiver instead of looking at how the Sun shines into a well to determine sun angle. We will use 30 meter measuring tapes to measure distance instead of riding the 5,000 stadia from Syene to Alexandria by camel. We will then establish a simple ratio to measure the circumference of the Earth!

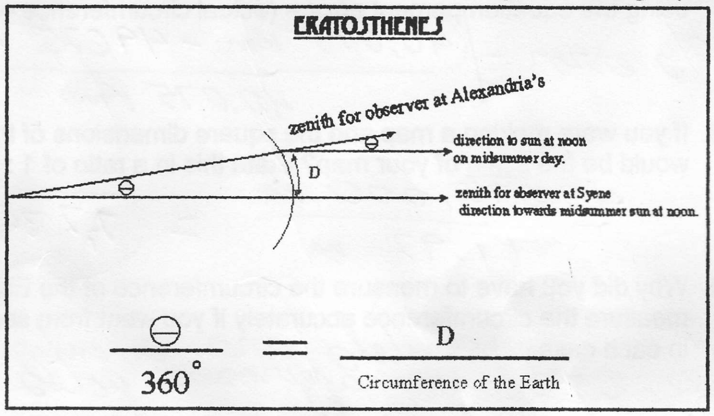

Image from http://www.astro.cornell.edu/academics/courses/astro2201/eratosthenes.htm

19378
19365
00013

Materials

- GPS Receiver
- 30 meter measuring tape
- Scientific calculator
- Pencil

Procedure and Questions to Answer

1. Divide into groups of 4 students.
2. Obtain a GPS receiver for your group. Make sure that this GPS receiver is set to decimal degrees for purposes of measuring latitude and longitude. Makes sure that its method of calculating distance is set to the metric system (meters/kilometers). I will run through how to do this with the class.
3. Obtain a tape measure for your group. You will be measuring using the metric system. Do not use the inches side! Ask me if you don't know which is which.
4. Proceed to the Oak Grove JV softball field.
5. Find first base on the JV softball field. Record the latitude and longitude at first base. Latitude: 34.19365° N

   Longitude: 118.18006° W
6. Measure the distance from first base to second base on the JV softball field. 17.92 meters
7. Record the latitude and longitude at second base on the JV softball field. Latitude: 34.19378° N

   Longitude: 118.18006° W
8. Subtract your latitude at first base from your latitude at second base.

Latitude at second base: 34.19378° – Latitude at first base: 34.19365° = 0.00013° ?

9. Write an equation below that will help you to solve for the circumference of the Earth. *Hint*: Think ratios and see graphic above!

$$\frac{\text{angle measured } (\theta)}{360^\circ} = \frac{\text{Distance Measured } (D)}{\text{Circumference of Earth } (x)}$$

10. Let the variable x in your equation be the circumference of the Earth. Solve for x **without adding in the numbers yet**!

angle measured (θ) • Circ. of Earth (x) = Distance (D) • 360°

$$\text{Circ of Earth } (x) = \frac{\text{Distance } (D) \cdot 360^\circ}{\text{angle measured } (\theta)}$$

From Eratosthenes to Global Positioning Systems: Calculating the Size of the Earth

Geology 1P — Mr. Traeger

11. Using the equation that you just found in number 10, add the numbers in with units and solve for x, the circumference of the Earth. If you have the right equation, your units of degrees will cancel each other out and you will be left with an answer in meters. Convert from meters to kilometers, knowing that there are 1,000 meters in 1 kilometer. State your answer for the circumference of Earth in kilometers! Show work!

$X = \frac{17.92\ m \cdot 360^\circ}{0.00013^\circ} = \frac{6451.2}{0.00013} = 49624615\ m \left(\frac{1\ Km}{1000\ m}\right) = 49625\ Km$

12. The established value for the circumference of the Earth is 40,075 km. Determine your percentage error below by using the equation percent error = (actual circumference – your answer / actual circumference) x 100. Show work!

$\%\ error = \left|\frac{(40,075\ Km - 49625\ Km)}{40,075\ Km} \times 100\right| = 23.8\%$ Why GPS error?

13. If you were making a map and the square dimensions of the map match the distance from first to second base, what would be the scale of your map? State this in a ratio of 1 : your number. Show work!

$\frac{40,075,000\ m}{17.92\ m} = 2,236,328 : 1$

14. Why did you have to measure the circumference of the Earth going from first to second base? Why could you not measure the circumference accurately if you went from second to third base? *Hint*: Think about the direction of travel in each case.

Parallels are evenly spaced. Merdians get closer as you approach poles. 1° lat = 111 Km = 69 miles = 60 nm

1° lon = 111 Km (Cos(lat°))

$111\ Km \cdot Cos(34^\circ) = 92\ Km$

Use the known value of Earth's circumference of 40,075 km to do the following calculations.

15. What is the radius of the Earth in kilometers? Find this by using the equation for circumference that you learned in geometry. Show work!

$C = 2\pi r \therefore r = \frac{C}{2\pi} = \frac{40,075\ Km}{2\pi} = 6378\ Km$

16. The Earth, if it were a perfect sphere, has a surface area that can be found by using the equation: surface area of a sphere = $4*\pi*r^2$. What is the surface area of the Earth in square kilometers? Show work!

$4\pi(6378^2\ Km^2) = 511,185933\ Km^2 = 5.11185933 \times 10^8\ Km^2$

17. The Earth, if it were a perfect sphere, has a volume that can be found using the equation: volume of a sphere = $4/3*\pi*r^3$. What is the volume of the Earth in cubic kilometers? Show work!

$\frac{4}{3} \cdot \pi \cdot (6378^3\ Km^3) = 1.09 \times 10^{12}\ Km^3$

18. Convert your answer in number 17 to units of cubic meters by multiplying your answer times 1 billion = 1,000,000,000 = 1×10^9. Show work!

$1.09 \times 10^{12}\ Km^3 \cdot 1 \times 10^9 = 1.09 \times 10^{21}\ m^3$

19. The mass of the Earth is 5.97×10^{24} kilograms. Knowing this and volume of the Earth found in # 18, calculate the Earth's average density in kilograms/cubic meter. Show work!

$D = \frac{m}{V} = \frac{5.97 \times 10^{24}\ Kg}{1.09 \times 10^{21}\ m^3} = 5493\ \frac{Kg}{m^3}$

20. Divide your answer for #19 by 1,000. This will convert your answer for density to units of grams/cubic centimeter. Show work!

$\frac{5493}{1000} = 5.493\ \frac{g}{cm^3} = \frac{g}{ml}$

21. Water has a density of 1 g/cm³. How many times heavier than water is the average density of Earth? Would Earth sink or float if you put it in water? Show work.

$\frac{5.493\ \frac{g}{cm^3}}{1\ \frac{g}{cm^3}} = 5.493 \times$ heavier $\therefore$ sink

22. Why is it impossible for Earth to be a perfect sphere? *Hint*: Think about what the Earth does every 24 hours and then relate this idea to the spin cycle in your washing machine at home.

Inertia will cause Earth to bulge out in the center, therefore creating an oblate spheroid.