ÅElton John

Rocket Science 101

Does conservation of energy really work?

The plan

- Launch a rocket of known mass at V_0
- Measure how high it goes
- Compare to calculations
- Analyze differences

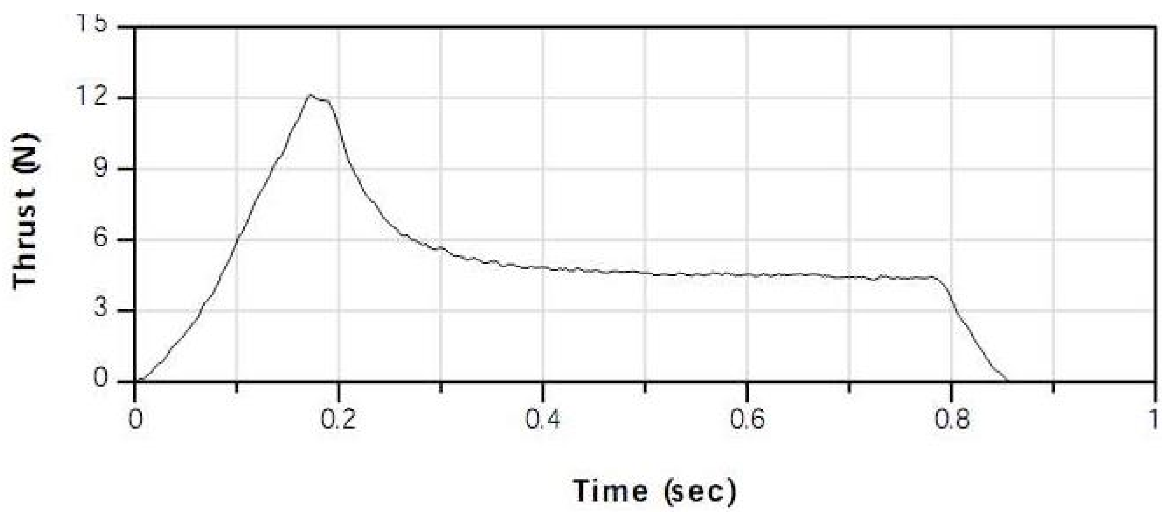
Estes rockets

- Airframe: Tube with fins & nose cone
- Engine: Solid propellant, fast burn
- Recovery: Ejected parachute

Engine characteristics

- Cardboard cylinder 17X70 mm
- Solid propellant casting
- Electric ignition with hot wire
- Delayed charge ejects chute

- Thrust(t) imparts impulse to rocket



Engine impulse

- Impulse = I (can be calculated from thrust curve)
- Velocity imparted V = I/M
  - Where I is the impulse imparted, and
  - M is the rocket mass (assumed constant)

How high will the rocket fly?

At launch, the rocket is essentially on the ground with
P.E. = 0, but
K.E. = $\mathbf{M} V^2/2$.

At maximum height H_{max} the rocket has zero velocity, hence
K.E. = zero,
but P.E. = $\mathbf{M} g H_{max}$

P.E. + K.E. at launch = $0 + \mathbf{M} V^2/2$
P.E. + K.E. at maximum height = $\mathbf{M} g H_{max} + 0$

Derivation continued

Setting these two equal gives
$\mathbf{M}\ V^2/2 = \mathbf{M}\ g\ H_{max}$
or $H_{max} = V^2/2g$

But V can be obtained from the previous slide, namely
$V = \mathbf{I/M}$
hence $V^2 = (\mathbf{I/M})^2$

Substituting this expression for V^2 into the expression for H_{max} one gets:
$H_{max} = (\mathbf{I/M})^2 /2g$

How long to max height?

- $V = V_0 - gt$
- At maximum height, $V = 0$
- Hence time to max height can be obtained by $0 = V_0 - gt$
- Or $t_{max} = V_0 / g$

Letôs put in some numbers

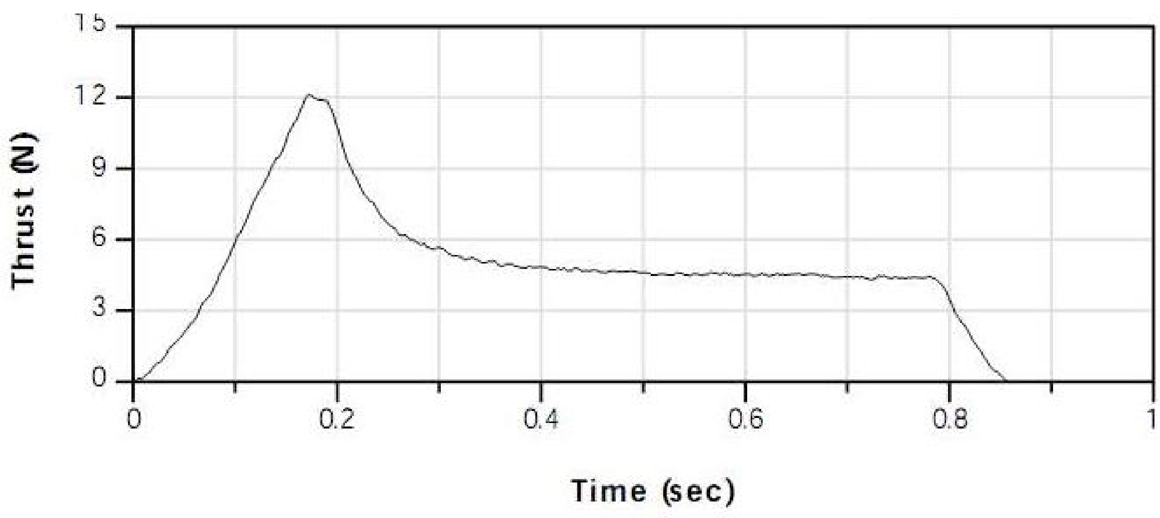
- What do we need to calculate H_{max} and t_{max}?
  - Engine impulse, I (Nt-secs)
  - Rocket mass, M (kg)
  - g (if you donôt know it now...)

How do we get impulse?

- The hardest way
  - Set up a fast acting force transducer and digital recorder; measure thrust vs time; integrate

How do we get impulse?

- The hard way
  - Numerically integrate the Estes thrust curve



How do we get impulse?

- The hardest way
  - Set up a fast acting scale and digital recorder; measure thrust vs time; integrate
- The hard way
  - Numerically integrate the Estes thrust curve
- The easy way
  - Look it up in the Estes tables

Estes engine specs

Prod. No.	Engine Type	Total Impulse	Time Delay	Max. Lift Wt.		Max. Thrust		Thrust Duration	Initial Weight		Propellant Weight	
		N-sec	Sec.	Oz.	g	Newtons	Lbs.	Sec.	Oz.	g	Oz.	g
SINGLE STAGE ENGINES (GREEN LABEL)												
1502	1/4A3-3T	0.625	3	1.0	28	4.9	1.1	0.25	0.20	5.6	0.03	0.85
1503	1/2A3-2T	1.25	2	2.0	57	8.3	1.9	0.3	0.20	5.6	0.06	1.75
1507	A3-4T	2.50	4	2.0	57	6.8	1.5	0.6	0.27	7.6	0.12	3.50
1511	A10-3T	2.50	3	3.0	85	13.0	2.9	0.8	0.28	7.9	0.13	3.78
1593	1/2A6-2	1.25	2	2.0	57	8.9	2.0	0.3	0.53	15.0	0.06	1.56
1598	A8-3	2.50	3	3.0	85	10.7	2.4	0.5	0.57	16.2	0.11	3.12
1601	B4-2	5.00	2	4.0	113	13.2	3.0	1.1	0.70	19.8	0.29	8.33
1602	B4-4	5.00	4	3.5	99	13.2	3.0	1.1	0.74	21.0	0.29	8.33
1605	B6-2	5.00	2	4.5	127	12.1	2.7	0.8	0.68	19.3	0.22	6.24
1606	B6-4	5.00	4	4.0	113	12.1	2.7	0.8	0.71	20.1	0.22	6.24
1613	C6-3	10.00	3	4.0	113	15.3	3.4	1.6	0.88	24.9	0.44	12.48
1614	C6-5	10.00	5	4.0	113	15.3	3.4	1.6	0.91	25.8	0.44	12.48
1622	C11-3	10.00	3	6.0	170	22.1	4.9	0.8	1.14	32.2	0.39	11.00
1623	C11-5	10.00	5	5.0	142	22.1	4.9	0.8	1.18	33.3	0.39	11.00
1666	D12-3	20.00	3	14.0	396	32.9	7.4	1.6	1.49	42.2	0.88	24.93
1667	D12-5	20.00	5	10.0	283	32.9	7.4	1.6	1.52	43.1	0.88	24.93
1673	E9-4	30.00	4	15.0	425	25.0	5.6	2.8	2.00	56.7	1.27	35.80
1674	E9-6	30.00	6	12.0	340	25.0	5.6	2.8	2.00	56.7	1.27	35.80

Estes engine specs

Prod. No.	Engine Type	Total Impulse	Time Delay	Max. Lift Wt.		Max. Thrust		Thrust Duration	Initial Weight		Propellant Weight	
		N-sec	Sec.	Oz.	g	Newtons	Lbs.	Sec.	Oz.	g	Oz.	g
SINGLE STAGE ENGINES (GREEN LABEL)												
1502	1/4A3-3T	0.625	3	1.0	28	4.9	1.1	0.25	0.20	5.6	0.03	0.85
1503	1/2A3-2T	1.25	2	2.0	57	8.3	1.9	0.3	0.20	5.6	0.06	1.75
1507	A3-4T	2.50	4	2.0	57	6.8	1.5	0.6	0.27	7.6	0.12	3.50
1511	A10-3T	2.50	3	3.0	85	13.0	2.9	0.8	0.28	7.9	0.13	3.78
1593	1/2A6-2	1.25	2	2.0	57	8.9	2.0	0.3	0.53	15.0	0.06	1.56
1598	A8-3	2.50	3	3.0	85	10.7	2.4	0.5	0.57	16.2	0.11	3.12
1601	B4-2	5.00	2	4.0	113	13.2	3.0	1.1	0.70	19.8	0.29	8.33
1602	B4-4	5.00	4	3.5	99	13.2	3.0	1.1	0.74	21.0	0.29	8.33
1605	B6-2	5.00	2	4.5	127	12.1	2.7	0.8	0.68	19.3	0.22	6.24
1606	B6-4	5.00	4	4.0	113	12.1	2.7	0.8	0.71	20.1	0.22	6.24
1613	C6-3	10.00	3	4.0	113	15.3	3.4	1.6	0.88	24.9	0.44	12.48
1614	C6-5	10.00	5	4.0	113	15.3	3.4	1.6	0.91	25.8	0.44	12.48
1622	C11-3	10.00	3	6.0	170	22.1	4.9	0.8	1.14	32.2	0.39	11.00
1623	C11-5	10.00	5	5.0	142	22.1	4.9	0.8	1.18	33.3	0.39	11.00
1666	D12-3	20.00	3	14.0	396	32.9	7.4	1.6	1.49	42.2	0.88	24.93
1667	D12-5	20.00	5	10.0	283	32.9	7.4	1.6	1.52	43.1	0.88	24.93
1673	E9-4	30.00	4	15.0	425	25.0	5.6	2.8	2.00	56.7	1.27	35.80
1674	E9-6	30.00	6	12.0	340	25.0	5.6	2.8	2.00	56.7	1.27	35.80

Letôs get specific

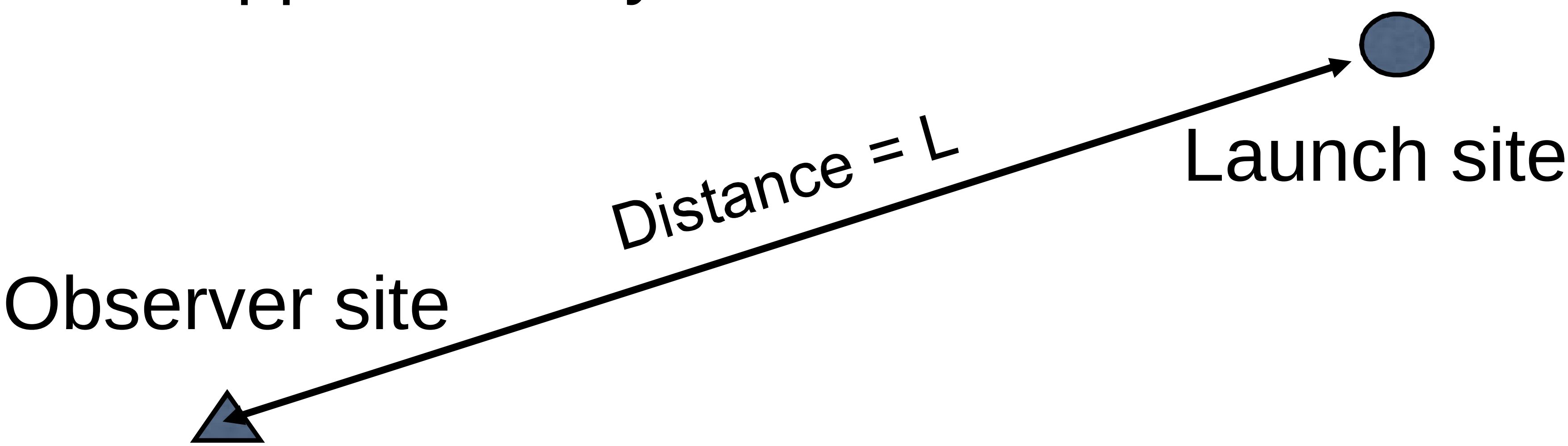
- We will use a rocket with mass 83 or 68 g
- We will use an A8-3 engine, I = 2.5 Nt-secs
- H_{max} = (**I/M**)^2 /2g
  - How high will it go?
- If V_0 = I/M, and t_{max} = V_0 / g
  - How long to maximum height?

Some things to think about

- If we used a more powerful engine, say B4-2 with I = 5 Nt-secs, or C6-3 with I = 10 Nt-secs, how high will this rocket go?
  - Do you think you could see it at burn out?
- What are the important assumptions used in this model?

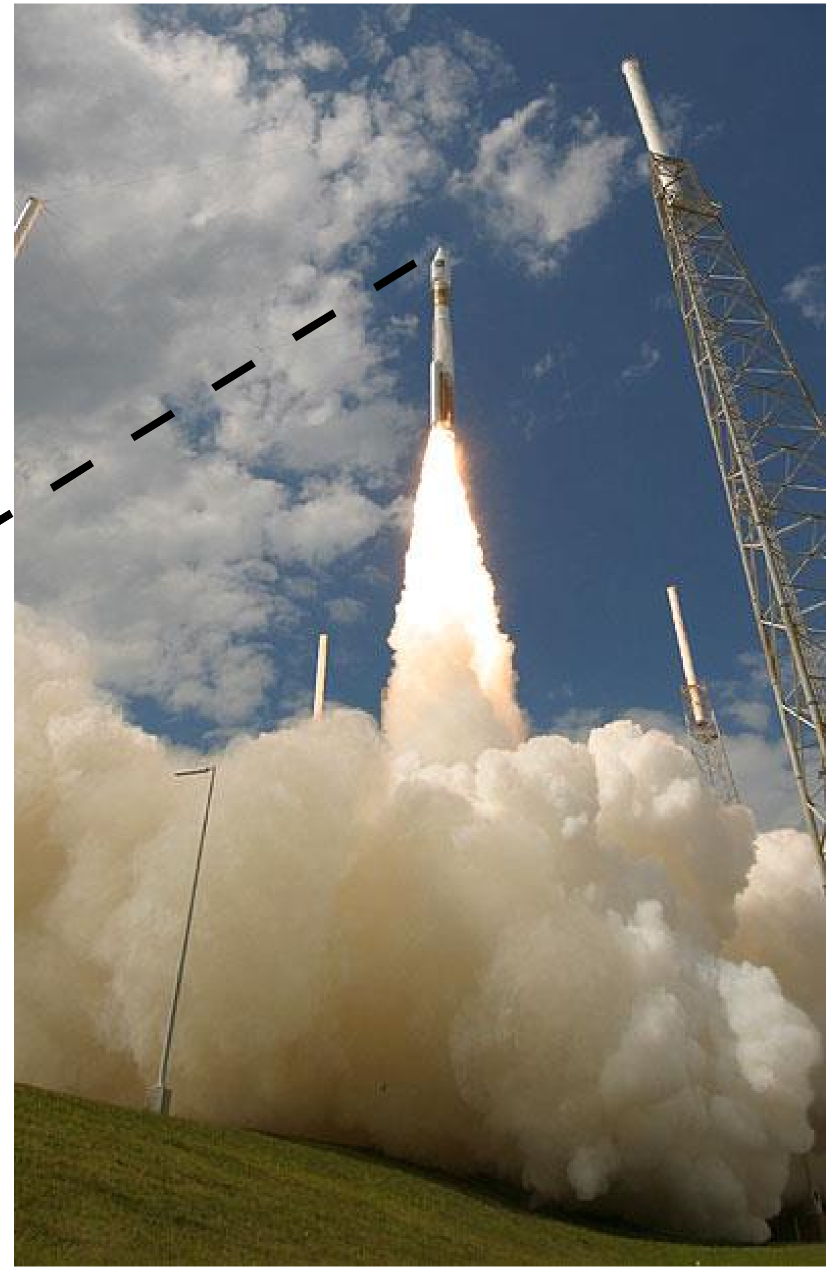
Off to the field

- How do we measure the height of the trajectory?
- Observer positions clustered as far as practical from launch site all at approximately the same distance L



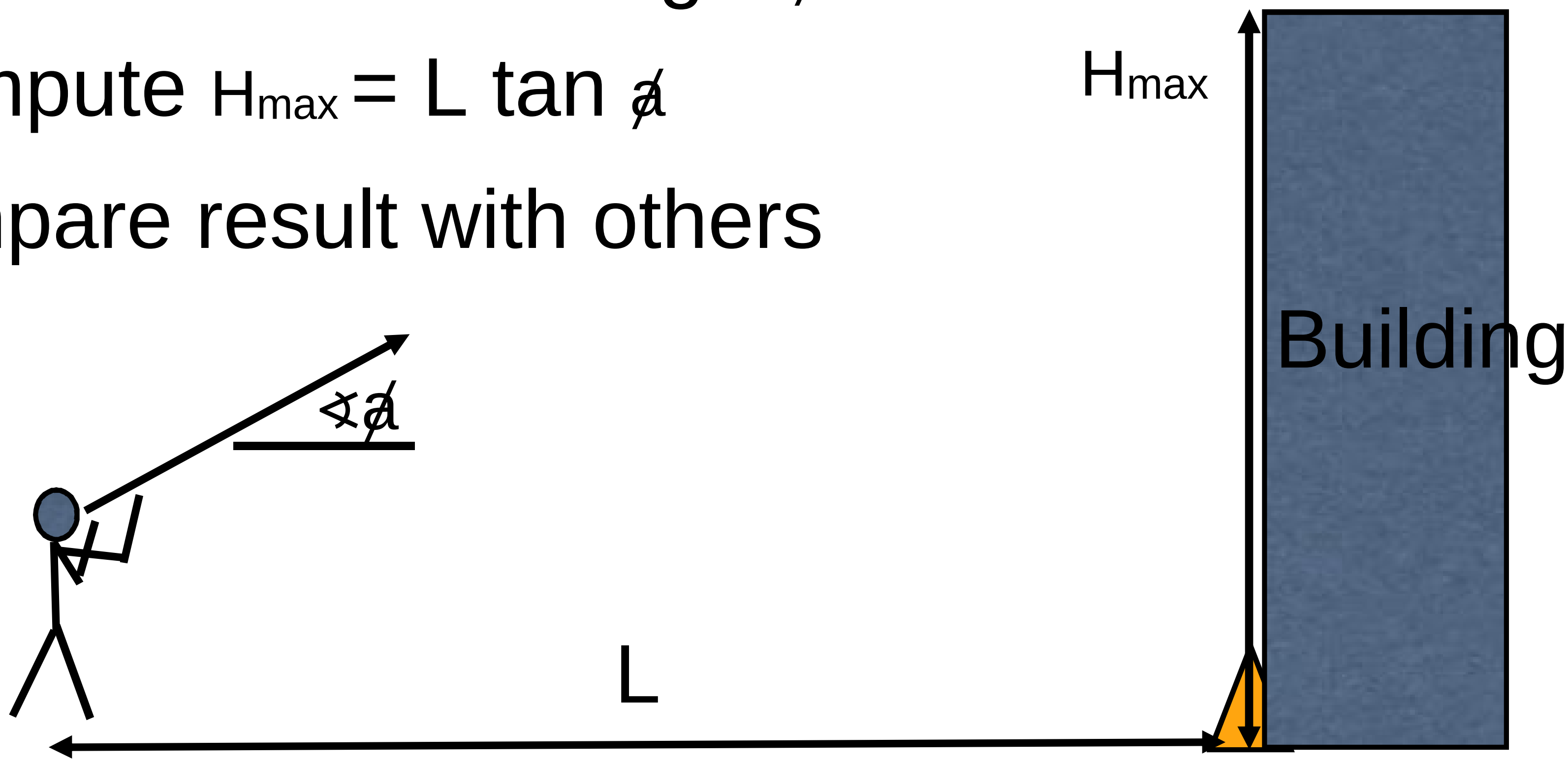
Plan View

The general idea



Trial run

- Sight on the upper edge of the building
- Measure *elevation* angle α
- Compute $H_{max} = L \tan \alpha$
- Compare result with others

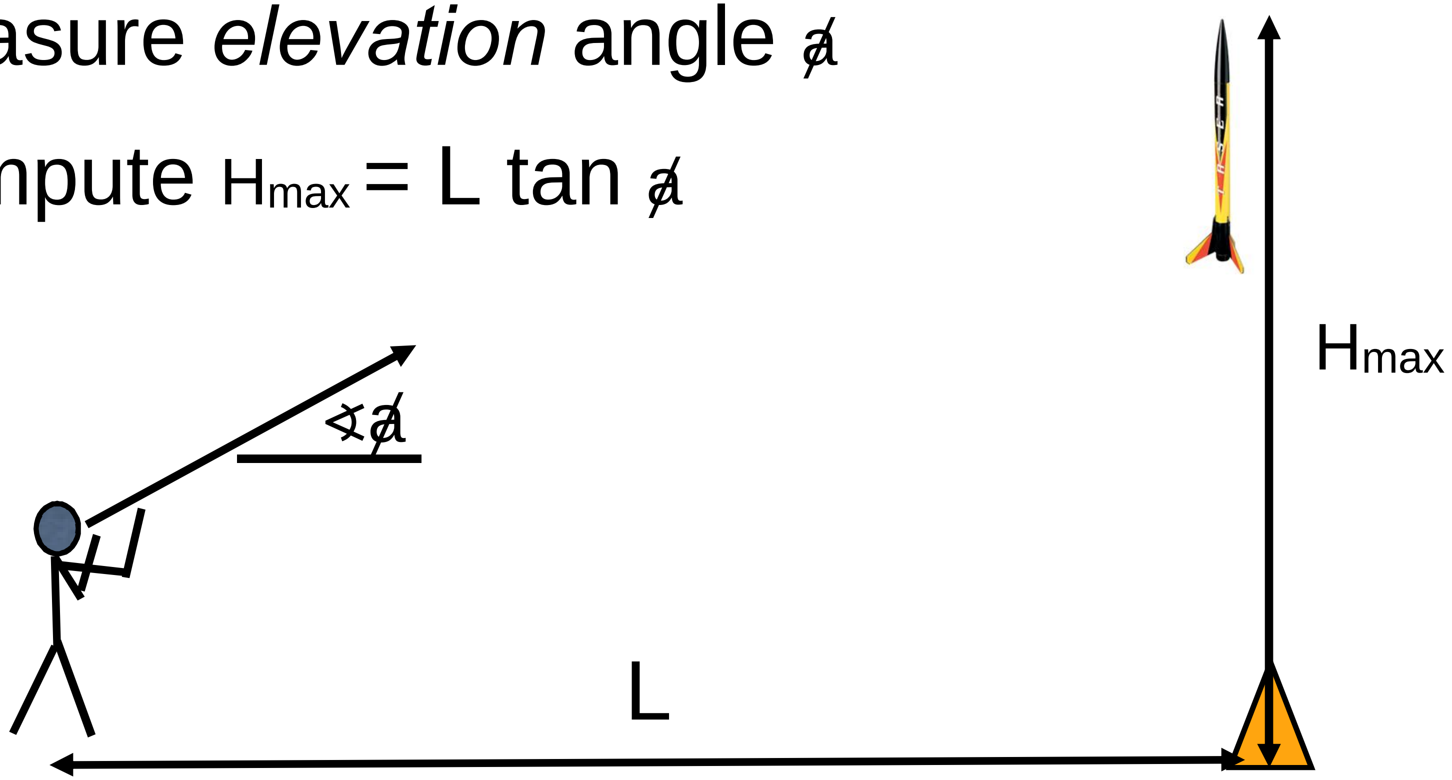


Trial run

- Our meter stick angle-measuring device is not as accurate as we would like it to be
- And it takes some practice to use it well
- To get some experience with this technique try something simple and stationary: Height of this building or the elevation of the Moon

Observer duties

- Track the rocket to its max height with meter stick
  - Measure *elevation* angle α
  - Compute $H_{max} = L \tan \alpha$



Data log

Observers

L = ft in = meters

Run #	α	tan α	H

Why so far from the launch?

- ÅImagine you were excitingly close to the launch such that the angle measured was 80°
- ÅCalculate the difference in computed height for a ±2° error
- ÅRepeat the calculation for a measured angle of 20°

Estimating errors in a̸

- It would be nice to make multiple measurements of a̸ and note the dispersion
  - But the uncooperative rocket won't stand still and allow many sightings!
- So what do we do?
  - Have several independent observers take sightings from the same spot
  - Then study the dispersions in their numbers to get mean and standard deviation

Past experiments tend to have lots of scatter in the data

- Maybe it has something to do with the measurement
  - We really only measure 2 things: L and α
  - How well do we measure them?
  - How do errors make a difference?
- First we'll look into the effect of measurement errors on the thing we are trying to know, H_{max}

Sensitivity

- H_{max} = L tan a̸
- Neither L nor a̸ are measured exactly
  - How much difference does that make to H?
  - In other words, how sensitive is H to errors in L and a̸ ?
- H is a function of L and a̸
  - æH is some function of æL and æa̸
    - Where æL and æa̸ are the errors in the measurements of those two quantities
- Letô s get ratios: æH/æL and æH/æa̸

Sensitivity continued

Å What is the limit of æH/æL as æL -> 0?

  Å Itôs the derivative dH/dL!

Å So for very small æL, æH/æL ≈ dH/dL

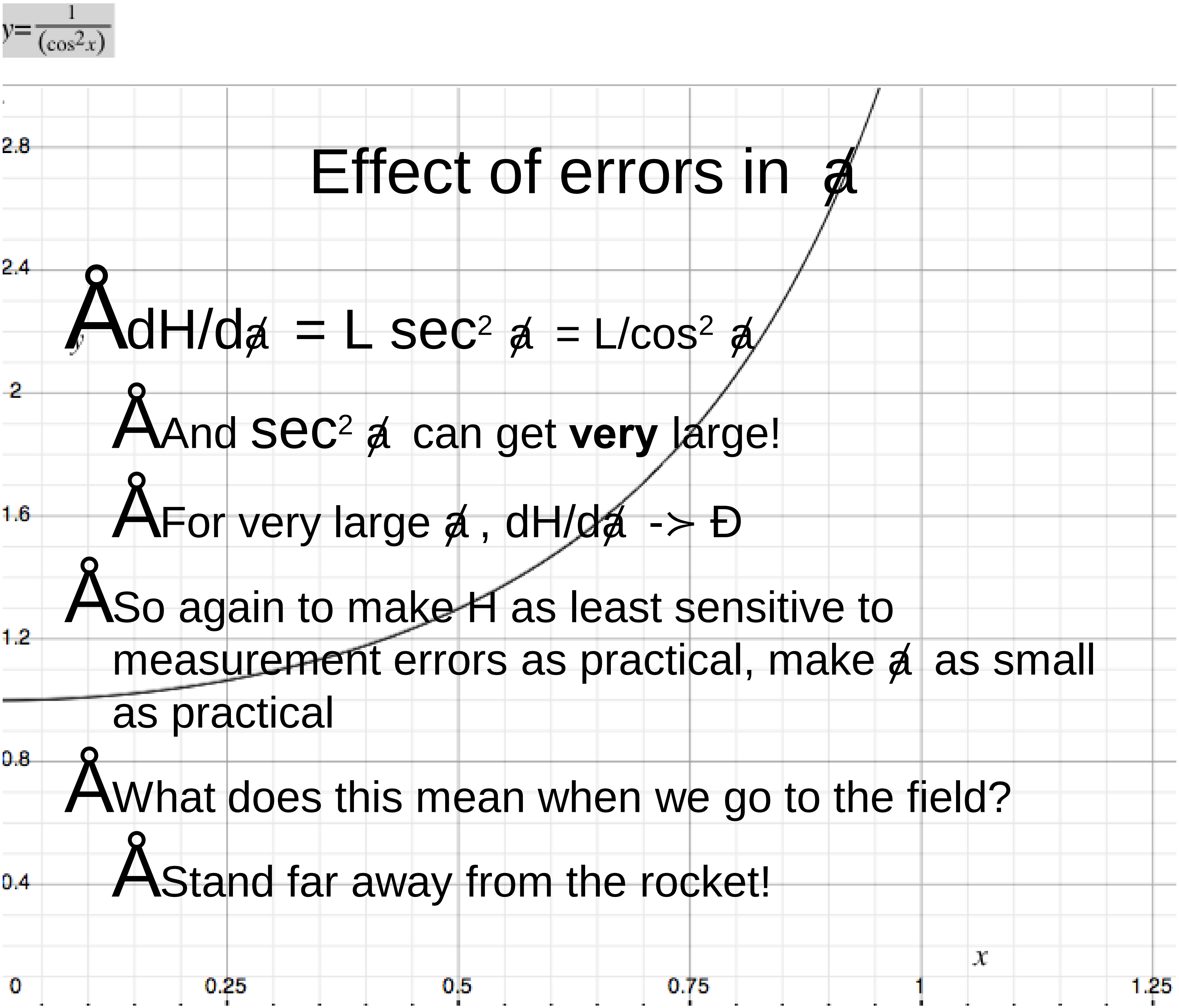
  Å Hence æH ≈ dH/dL Å æL

Å From H_{max} = L tan ø , dH/dL = tan ø

Å To make the error in H, æH, as **insensitive** as possible to errors in L, æL, what do we do?

  Å We make tan ø as small as practical

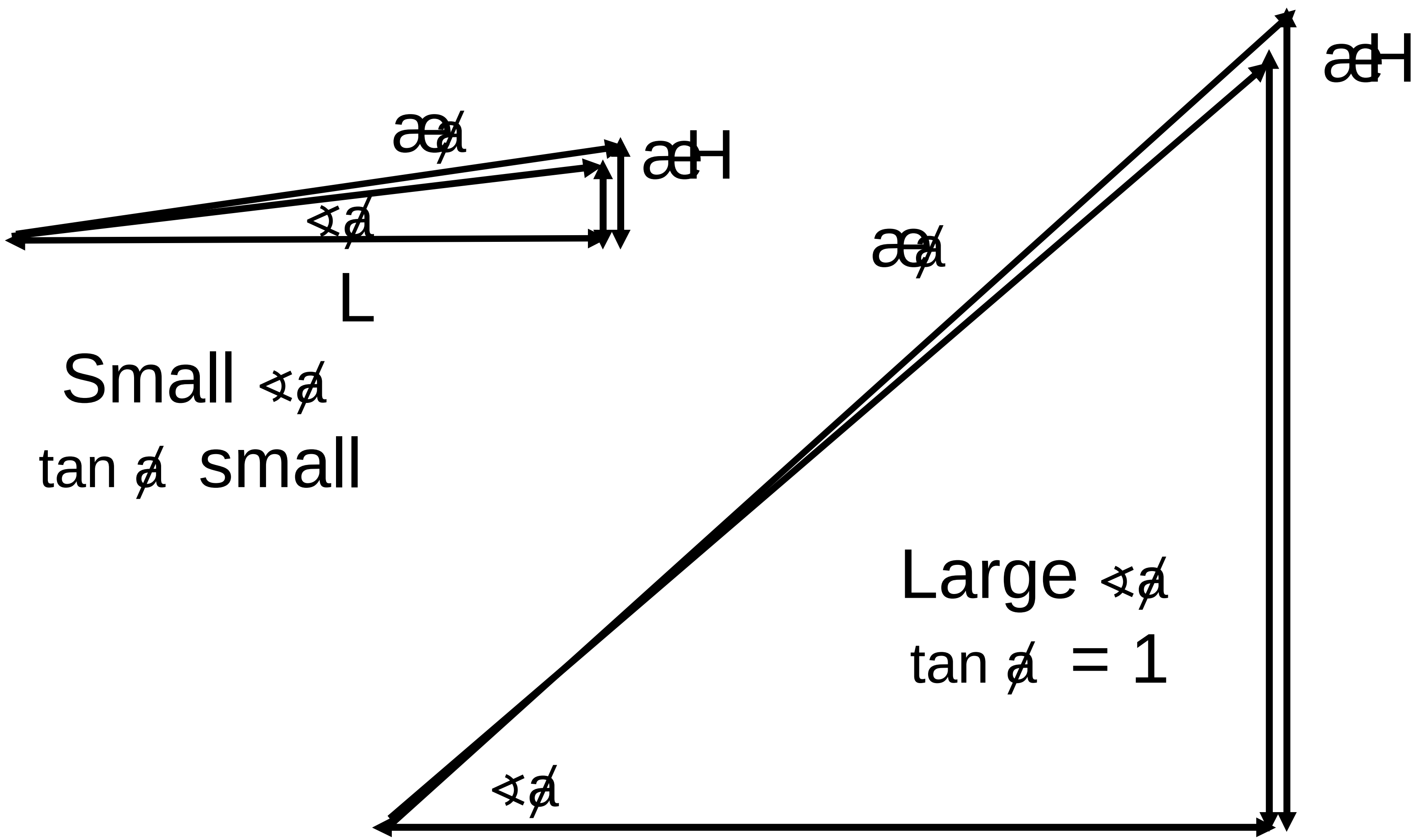
  Å So we make ø as small as possible



Effect of errors in θ

- $dH/d\theta = L \sec^2 \theta = L/\cos^2 \theta$
  - And $\sec^2 \theta$ can get **very** large!
  - For very large θ, $dH/d\theta \to \infty$
- So again to make H as least sensitive to measurement errors as practical, make θ as small as practical
- What does this mean when we go to the field?
  - Stand far away from the rocket!

Geometric interpretation



Minimize sensitivity to measurement errors

- To make the error in H, ΔH, as **insensitive** as possible to errors in L, ΔL, what do we do?
  - We make tan θ as small as practical
  - So we make θ as small as possible
- To make theta as small as possible, we get as far back from the launch site as practical
  - I know, that's less exciting!

The plan

- Groups of 2-3 students
- Pick a place to make your measurements
- Three students make the necessary horizontal measurement, L in the figure
- Others make angle measurements
  - Do them independently and privately
- Switch and do it again

Some complications

- The mass of the rocket it not constant
  - Propellant mass is about 3 g ~ 5% of total
  - Rocket equation $\Delta v = v_e \ln \frac{m_0}{m_1}$
    - Where v_e is exhaust velocity and
    - m_0 & m_1 are initial and final masses
- Not a big effect for this size rocket

End of lecture

- Off to the field!!!

More complications

- Aerodynamics really works
  - Drag on the rocket is ~.08 Nt @ 50 m/s
    - Proportional to V^2
    - Max thrust ~ 10 Nt, weight ~ 0.5 Nt
- How well did you track the rocket?
- Was the rocket L meters away?
- How accurately did you measure ϕ ?
- Did you make computational errors?

How big are the measurement errors?

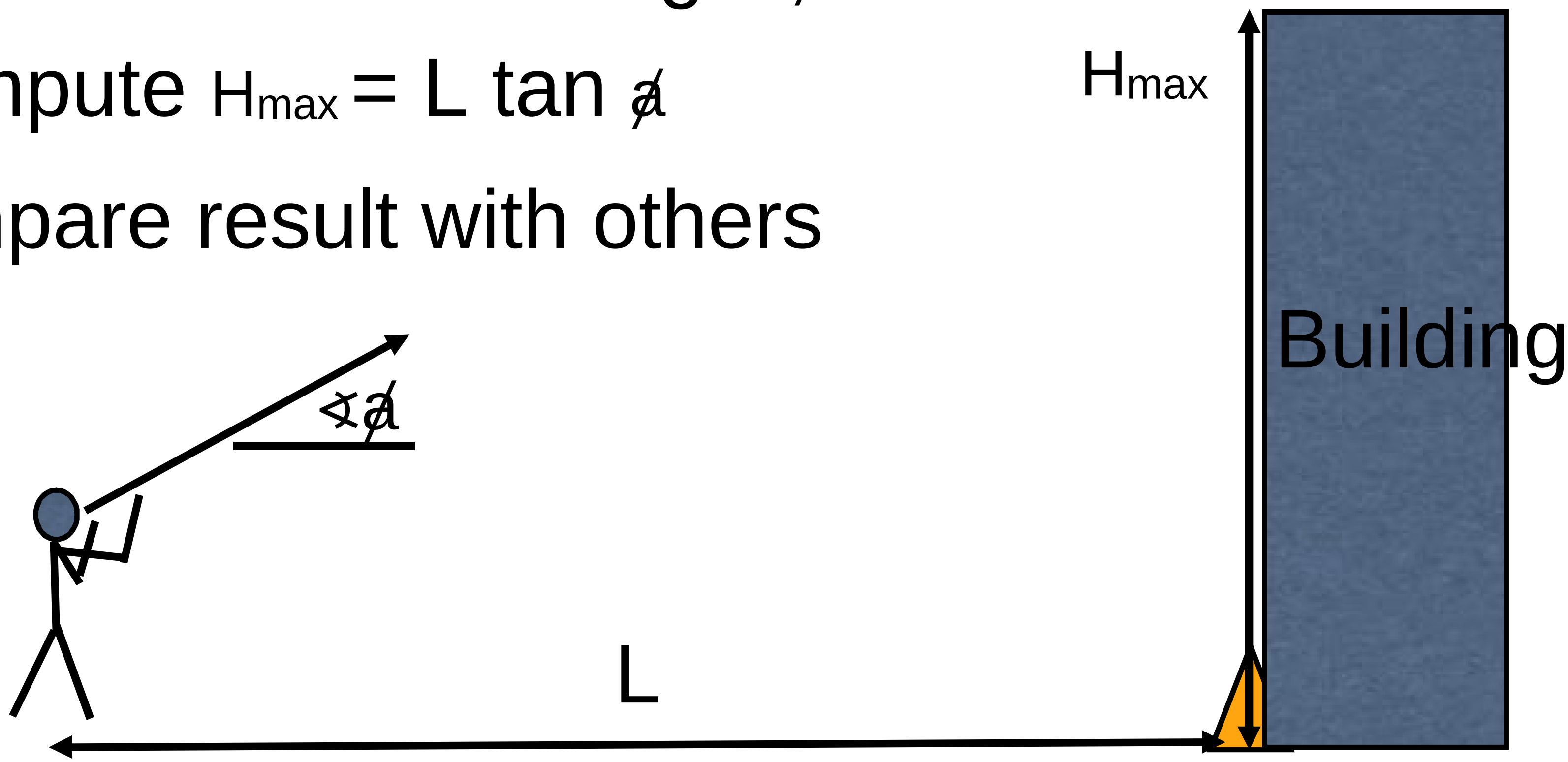
- Note that æH ≈ dH/dL ÅæL
  - And æH ≈ dH/da̸ Åæa̸
- But approximately how big are æL and æa̸ ?
- How do determine uncertainty in things we measure?
  - Measure them several times and note the differences
  - That works for L, make multiple tries and see how they differ
  - How much would you expect æL to be?

Let's try some realistic numbers

- $\Delta H = dH/d\theta \cdot \Delta\theta = (L/\cos^2\theta) \cdot \Delta\theta$
  - What is a reasonable value of $\Delta\theta$?
    - Try $\Delta\theta = 0.1$ radians
- The smallest $dH/d\theta$ can be is L (when $\theta = 0$)
- So even for small θ, $\Delta H \approx L\,\Delta\theta$ or about 0.1L
  - For larger θ, it's even worse
- This may be the whole reason results are scattered

Trial run

- Sight on the upper edge of the building
- Measure *elevation* angle α
- Compute $H_{max} = L \tan \alpha$
- Compare result with others



What do we do with the observations?

Observer Grp	L	Launch 1 a̸	Launch 2 a̸	Launch 3 a̸
A				
B				
C				
D				
E				
F				
Mean				
ů				
Mean H = L tan a̸				
L tan (a̸ ± ů)				

Data reduction

- Back in the classroom, we will crunch the results
- Letôs see if we can get a best estimate of the actual max height and compare that to calculations